Energy Detection of DVB-T Signals Against Noise Uncertainty

Ming Jin, Qinghua Guo, Jun Tong, Jiatao Xi, and Youming Li

Abstract—In this letter, we consider the spectrum sensing of digital video broadcasting-terrestrial (DVB-T) signals with energy detection (ED). ED delivers a superior performance among low computational complexity detectors, but it is vulnerable to noise uncertainty. By exploiting the pilot periodicity in DVB-T signals, we propose a low complexity noise power estimator which works in multipath channels, and does not require time and frequency synchronization. Simulations show that, with the noise power estimator, ED significantly outperforms the pilot correlation-based detection which is often used in DVB-T spectrum sensing.

Index Terms—Energy detection, noise uncertainty, spectrum sensing.

I. INTRODUCTION

COGNITIVE radio (CR) has been recognized as one of the promising technologies to alleviate the issue of wireless spectrum shortage [1]. Recently, IEEE 802.22 has been developed for cognitive radio operating in television spectral bands [2]. This work focuses on spectrum sensing (a key functionality in CR [3]) of digital video broadcasting-terrestrial (DVB-T) signals.

Energy detector (ED) has attracted much attention due to its low computational complexity [4]. It does not need the knowledge of primary signals, and is capable of sensing any type of signals, e.g., orthogonal frequency division multiplexing (OFDM) signals. Ideal ED, which requires the knowledge of exact noise power, delivers a superior performance among low complexity detectors [5], [6]. However, the knowledge of the exact noise power is usually unavailable. To overcome this problem, it is often assumed that the exact noise power falls into a bounded range, and the upper bound is employed to replace the exact noise power in ED. This replacement leads to noise uncertainty, which severely degrades the performance of ED, i.e., ED is sensitive to noise uncertainty. An alternative approach is to use an estimated noise power in ED. A thorough analysis of the effect of the estimated noise power on the performance of ED has been given in [4]. The noise power can be estimated with noise-only samples. However, it is a challenging task if the presence of primary signals is unknown.

In sensing OFDM signals, such as DVB-T signals, correlation-based detectors were proposed by exploiting the correlation of cyclic prefix or pilot [7]–[12]. In these detectors, the exact noise power is replaced with the power of received signals (which may contain primary signals). The detectors are robust to noise uncertainty, but compared with the ideal ED, they exhibit considerable performance degradation (about 7 ~ 15 dB) as shown in [12].

In this letter, instead of using correlation-based detectors for DVB-T spectrum sensing, we employ ED (with an accurate estimate of noise power) due to its low complexity and superior performance. By exploiting the pilot periodicity in the DVB-T signals, we propose a low complexity noise power estimator which is able to obtain an accurate noise power estimate from the received signals regardless of the presence of the primary DVB-T signals. The influence of carrier frequency offsets (CFOs) is eliminated in the noise power estimator, and frequency synchronization is not required. Due to the pilot periodicity in DVB-T, time synchronization is not needed either. Moreover, because the pilot periodicity still remains over multipath channels, multipath channels have no impact on the performance of the proposed noise power estimator (assuming that the multipath channels have the same energy). Simulation results show that the proposed noise power estimator enables ED to achieve a reliable detection performance.

II. SIGNAL MODEL AND EXISTING DETECTORS

A. Signal model

Let \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) represent the absence and presence of primary (DVB-T) signals, respectively. The received signal \( r(k) \) at a secondary user is given by

\[
r(k) = \eta e^{j2\pi f \Delta kj} x(k) + w(k)
\]

where \( k = 0, 1, \cdots, K - 1 \) is the time index; \( \eta = 0 \) under \( \mathcal{H}_0 \) and \( \eta = 1 \) under \( \mathcal{H}_1 \); \( \Delta f \) denotes the unknown CFO; \( w(k) \) denotes the independent and identically distributed circularly symmetric complex Gaussian white noise with mean zero and variance \( \sigma_w^2 \), i.e., \( w(k) \sim \mathcal{CN}(0, \sigma_w^2) \); and the DVB-T signal \( s(k) \) arrives at the secondary user through a length-\( M \) multipath channel \( h(k) \), resulting in the received primary signal \( x(k) \).

The DVB-T signal \( s(k) \) consists of pilot and data signals. Fig. 1 illustrates the grid of scattered pilot and data subcarri-
ers\(^1\). The time domain signal \(s(k)\) can be represented as
\[
s(k) = s_p(k) + s_d(k)
\]
where \(s_p(k)\) and \(s_d(k)\) denote the pilot signal and the data signal, respectively. For a sufficiently large number of subcarriers, both \(s_p(k)\) and \(s_d(k)\) are approximately Gaussian distributed. As shown in Fig. 1, \(s_p(k)\) has a repetition period of four OFDM symbols, while \(s_d(k)\) has no periodicity [11], [13]. Assuming that the sampling frequency is an integer multiple \(s\) of \(2\) and \(4\) respectively. For a sufficiently large number of subcarriers, the correlation coefficient in (15), however, the method is still applicable when \(s\) has a repetition period of four OFDM symbols, while \(s\) has no periodicity [11], [13].

![Fig. 1. Scattered pilot symbol grid of DVB-T signals.](image)

\[x(k) = x_p(k) + x_d(k)\]
\[
= \sum_{m=0}^{M-1} h(m)s_p(k - m) + \sum_{m=0}^{M-1} h(m)s_d(k - m).
\]

It can be easily shown that \(x_p(k)\) also has a period of \(N\), i.e., \(x_p(k) = x_p(k + N)\), while \(x_d(k)\) has no periodicity. In addition, both \(x_p(k)\) and \(x_d(k)\) follow Gaussian distributions. Let \(\sigma_p^2\) and \(\sigma_d^2\) denote the power of \(x_p(k)\) and \(x_d(k)\), respectively, i.e., \(x_p(k) \sim \mathcal{CN}(0, \sigma_p^2)\) and \(x_d(k) \sim \mathcal{CN}(0, \sigma_d^2)\). Define \(\alpha\) as the power ratio of all data subcarriers to all pilot subcarriers in \(s(k)\). According to (3), we have \(\sigma_d^2 = \alpha \sigma_p^2\). With (1) and (3), the received signal \(r(k)\) can be rewritten as
\[
r(k) = \eta_r \rho_{\Delta} e^{j2\pi\Delta_f k} x_p(k) + \eta_w w(k),
\]
where \(x_p(k)\), \(x_d(k)\) and \(w(k)\) are independently Gaussian distributed. The purpose of this letter is to accurately estimate the power of the noise \(w(k)\) using \(r(k)\) without any knowledge of \(\eta\) (= 0 or 1) and \(\Delta_f\), thereby achieving reliable detection by employing ED.

**B. ED and TDSC-NP**

The test-statistic of ED is given by [5], [6]
\[
T_{ED} = \frac{1}{K} \sum_{k=0}^{K-1} |r(k)|^2.
\]

As the exact noise power \(\sigma_w^2\) is unknown, it is often assumed to be in a bounded range of \([\sigma_{w1}^2, \sigma_{w2}^2]\). The ED in worst-case employs \(\sigma_w^2\) to replace the exact noise power. This bounded worse behavior (BWB) model causes the noise uncertainty [4], which is defined as \(\beta = 10 \log_{10}(\sigma_{w2}^2/\sigma_{w1}^2)\) in dB.

\(^1\)The proposed method can be easily extended to exploit the continual pilots whose repetition period is one OFDM symbol.

\(^2\)The sampling frequency offset can impact on the proposed method through the correlation coefficient in (15), however, the method is still applicable when the effect of the sampling frequency offset is negligible.

In [11], TDSC-NP (time-domain symbol cross-correlation Neyman-Pearson) detector was proposed for sensing DVB-T signals, and the test-statistic is given by
\[
T_{TDSC-NP} = \frac{1}{K-N-1} \sum_{k=0}^{K-N-1} r(k)r^*(k + N)
\]
where the exact noise power \(\sigma_w^2\) is also required. As in many other correlation-based detectors, the unknown \(\sigma_w^2\) can be replaced with the estimated power of the received signal [8]
\[
\hat{\sigma}_w^2 = \frac{1}{K} \sum_{k=0}^{K-1} |r(k)|^2,
\]
i.e., the noise power and the noise-plus-primary-signal power are not distinguished. TDSC-NP is robust against noise uncertainty, but even with exact noise power, it suffers from considerable performance loss compared to the ideal ED as shown in [12] and in Section IV of this letter.

**III. NOISE POWER ESTIMATION FOR ED**

It is difficult to estimate the noise power \(\sigma_w^2\) with (4) due to the following reasons. The presence of the primary signal in \(r(k)\), i.e., \(\eta = 0\) or 1, is unknown. Even if we know that the primary signal is present, it is still difficult to estimate \(\sigma_w^2\) due to the interference from the data signal \(x_d(k)\) (in DVB-T, the power of \(x_d(k)\) is about 6.6 dB higher than \(x_p(k)\) [13]). In addition, the frequency and time synchronization is not assumed. Next, we will address this issue.

**A. Noise Power Estimator**

Let the correlation coefficient of \(r(k)\) at lag \(N\) be \(\rho_{\Delta}\), i.e.,
\[
\rho_{\eta} = \frac{1}{\sigma_r^2} E[|r(k)r^*(k + N)|]
\]
where \(E[\cdot]\) denotes the expectation operator and
\[
\sigma_r^2 = \eta_r \sigma_p^2 + \eta_w \sigma_d^2 + \sigma_w^2.
\]
It can be verified that
\[
E[|r(k)r^*(k + N)|] = \eta_r e^{-j2\pi\Delta_f N} \sigma_r^2.
\]
Hence, we have
\[
\rho_{\eta} = e^{-j2\pi\Delta_f N} \frac{\eta_r \sigma_r^2}{\eta_r \sigma_r^2 + \eta_w \sigma_d^2 + \sigma_w^2}.
\]
Let
\[
\rho_{\eta} = e^{j2\pi\Delta_f N} \rho_{\Delta}\.
\]
According to (11), \(\rho_{\eta}\) can also be represented as
\[
\rho_{\eta} = \frac{\eta_r \sigma_r^2}{\eta_r \sigma_r^2 + \eta_w \sigma_d^2 + \sigma_w^2},
\]
which together with \(\sigma_w^2 = \alpha \sigma_p^2\) gives the noise power
\[
\sigma_w^2 = [1 - (1 + \alpha)\rho_{\eta}] \sigma_p^2.
\]
The power of the received signal \(\sigma_r^2\) can be easily estimated with (7). The key to estimate \(\rho_{\eta}\).

With the received signal \(r(k)\), \(\rho_{\eta}\) can be estimated as
\[
\hat{\rho}_{\eta} = \frac{1}{(K - N) \sigma_r^2} \sum_{k=0}^{K-N-1} r(k)r^*(k + N).
\]
In order to deal with the phase induced by the unknown CFO, \(\hat{\rho}_{\eta}\) in (15) is separated into two parts as shown in (16) at the
the noise power can be estimated as

\[ \hat{\sigma}_w = |1 - (1 + \alpha)\hat{\rho}_n|\sigma_r^2. \]  

(18)

It is attractive that the proposed noise power estimator works no matter whether primary signals are present or not. We can take this advantage and use a sliding window approach to achieve an accurate estimate of the noise power. The estimate for the \( q \)th (current) sensing duration can be represented as

\[ \hat{\sigma}_w^2(q) = \frac{1}{Q} \sum_{q'=q-Q+1}^q \hat{\sigma}_{w,q'}^2 \]  

(19)

where \( Q = 1 \) previous sensing durations are used, and \( \hat{\sigma}_{w,q'}^2 \) denotes the estimated noise power using (18) for the \( q' \)th sensing duration.

B. Analysis of Estimation Bias

With (16) and (17), (18) can be rewritten as (20) at the bottom of this page where both \( \hat{\rho}_{n1} \) and \( \hat{\rho}_{n2} \) are the estimates of \( \rho_n \). It can be verified that both \( \hat{\rho}_{n1} \) and \( \hat{\rho}_{n2} \) are unbiased estimates of \( \rho_n \). Hence, from (20), the estimation bias of \( \hat{\sigma}_w^2 \) is given by

\[ E[\hat{\sigma}_w^2] - \sigma_w^2 = \eta \sigma_p^2 + \sigma_q^2 - 0.5(1 + \alpha)\sigma_p^2 \rho_n E[\cos \delta_1 + \cos \delta_2] \]  

\[ = (\eta \sigma_p^2 + \sigma_q^2)(1 - 0.5\text{E}[\cos \delta_1 + \cos \delta_2]). \]  

(21)

In noise only case (\( \eta = 0 \)), \( E[\hat{\sigma}_w^2] - \sigma_w^2 = 0 \), i.e., \( \hat{\sigma}_w^2 \) is an unbiased estimate. Otherwise, \( \hat{\sigma}_w^2 \) is biased toward overestimation. At high SNRs (signal-to-noise ratios), its bias is small because \( 0.5\text{E}[\cos \delta_1 + \cos \delta_2] \) approaches one [14]. It is interesting that low SNRs also lead to a small bias, because \( \sigma_p^2 + \sigma_q^2 \) (with fixed \( \sigma_r^2 \)) approaches zero. This is very useful for spectrum sensing in low SNR regime.

\[ \hat{\rho}_n = \frac{1}{(K - N)\sigma_r^2} \sum_{k=0}^{K-N-1} r(2k+1) r^*(2k+1+N) + \frac{1}{(K - N)\sigma_r^2} \sum_{k=0}^{K-N-1} r(2k) r^*(2k + N). \]  

(20)

\[ \hat{\sigma}_w^2 = \sigma_r^2 - (1 + \alpha)\sigma_r^2 \text{Re} \left( \frac{e^{i\delta_2}}{2} \sum_{k=0}^{K-N-1} r(2k+1) r^*(2k+1+N) \right) + \frac{e^{i\delta_1}}{2} \sum_{k=0}^{K-N-1} r(2k) r^*(2k + N) \].

C. Computational Complexity

The calculation of \( \hat{\sigma}_w^2 \) with (7) requires about \( K \) multiplications, which can be shared by the calculations of \( \alpha_1 \) and \( \alpha_2 \). Thus, the calculation of \( \hat{\rho}_n \) with (17) requires about \( K - N \) multiplications. The complexity of ED with noise power estimator is only about \( 2K - N \) multiplications.

IV. SIMULATION RESULTS

We employ the 2K mode DVB-T with 1/8 guard interval. The DVB-T signal has 2048 subcarriers, including 141 scattered pilot subcarriers, 45 continual pilot subcarriers and 343 null subcarriers. The sample number of each OFDM symbol is 2304 (= 2048 + 2048/8). The power ratio of each data subcarrier to each pilot subcarrier is 9/16, hence \( \alpha \approx 0.59 \) [13]. Considering that the repetition period of scattered pilot subcarriers is four OFDM symbols, we take a sensing duration as a time frame of eight OFDM symbols, i.e., \( K = 2N \). In addition, \( Q = 200 \) sensing durations are used for noise power estimation4. The CFO is set to be \( \Delta_f = 0.4/N \) (arbitrarily selected). We define the SNR as \( (\sigma_p^2 + \sigma_q^2)/\sigma_r^2 \).

Fig. 2 shows the root-mean-square-error (RMSE) and bias of the estimated noise power when \( \sigma_r^2 = 1 \) for (a) different SNRs with \( Q = 200 \) and (b) different values of \( Q \) with SNR = -20 dB.

The noise power level is stationary typically for a few minutes [15]. In the 2K mode of DVB-T, the time frame of eight OFDM symbols is about 2.5 milliseconds, and \( Q = 200 \) sensing durations take about 0.5 seconds, which is much less than a few minutes.
The performance of TDSC-NP-E is very close to that of TDSC-NP (with exact noise power). As for ED-E, an SNR of $-14.08$ dB is required to achieve the detection probability of 0.9. ED-E significantly outperforms the TDSC-NP with exact noise power and the ED with a very small noise uncertainty of $\beta = 0.1$ dB. In addition, it can be observed that the variation of the false alarm probability of ED-E is consistent with that the noise power estimation bias shown in Fig. 2(a).

V. CONCLUSIONS

In this letter, we have employed ED for DVB-T spectrum sensing by acquiring an accurate estimate of the noise power. Simulations have shown that ED with our noise power estimator significantly outperforms both correlation-based detectors (such as TDSC-NP) and the ED with BWB noise model.

REFERENCES


To keep the figure clear, we only show the performance of TDSC-NP for correlation-based detection. The comparisons of TDSC-NP with other correlation-based detectors can be found in [12].

This is the author’s version of an article that has been published in this journal. Changes were made to this version by the publisher prior to publication. The final version of record is available at http://dx.doi.org/10.1109/LCOMM.2014.2347278